SE / EXTC / III (Rev)
App. Maths - III
24/11/12

D : scan Oct.12 294
Con. 7340-12.

KR-3098

(3 Hours) [**Total Marks : 100**

N.B. (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** out of the remaining **six** questions.
(3) **Non-programmable** calculator are **allowed**.

1. (a) $A = \frac{1}{7}\begin{bmatrix} 2 & 6 & 3 \\ 6 & -3 & 2 \\ 3 & 2 & -6 \end{bmatrix}$ 5

Show that –
(i) $|A| = 1$
(ii) $\operatorname{adj} A = A'$

(b) Show that the Fourier cosine transform of– 5

$f(x) = \frac{1}{\sqrt{x}}$ is $\frac{1}{\sqrt{s}}$

(c) Show that $L^{-1} \tan^{-1}\left(\frac{a}{s}\right) = \frac{\sin(at)}{t}$ 5

(d) Show that $\int_0^\infty e^{-t} \sin\left(\frac{t}{2}\right) \sinh\left(\frac{\sqrt{3}\,t}{2}\right) dt \; \frac{\sqrt{3}}{2}$ 5

2. (a) If z is any non zero complex number show that– 6

$A = \frac{1}{\sqrt{2}\,|z|}\begin{bmatrix} z & -\bar{z} \\ z & \bar{z} \end{bmatrix}$ is a unitary matrix.

(b) Find the Fourier series expansion of $f(x) = x^2$ in $[0, 2\pi]$ 6

(c) Show that $\int_0^\infty \frac{\sin 2t + \sin 3t}{t e^t} dt = \frac{3\pi}{4}$. 8

3. (a) Find the half range sine series for $f(x) = \pi x - x^2$ in $[0, \pi]$, hence find $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}$. 6

(b) Show that $L(\cos at \cosh at) = \frac{s^3}{s^4 + 4a^4}$. 6

(c) If a, b, c are distinct real numbers such that $a + b + c \neq 0$. Show that the vectors (a, b, c), (b, c, a) and (c, a, b) are linearly independent. 8

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4. (a) Find the Fourier series expansion of – **6**

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ 0, & 0 < x < \frac{\pi}{2} \\ x - \frac{\pi}{2}, & \frac{\pi}{2} < x < \pi \end{cases}$$

(b) Find all the possible ranks of the matrix **6**

$$A = \begin{bmatrix} k & -3 & -3 \\ -3 & k & -3 \\ -3 & -3 & k \end{bmatrix}$$ where k is any real number.

(c) Find – (i) $L^{-1} \dfrac{se^{-\pi s}}{s^2+3s+2}$; (ii) $L\,[\,t.H(t-4) + t^2\,\delta(t-4)\,]$ **8**

5. (a) Solve the system of equation using the Gauss-Seidel method : **6**

$$\begin{aligned} 3x + 4y + 6z &= 6 \\ 2x + 5y + 4z &= 11 \\ 2x + y + z &= 5 \end{aligned}$$

(b) Find $L^{-1} \dfrac{s}{(s^2+2)^2}$ using the convolution theorem. **6**

(c) Show that $z\,[\cos(\alpha k)] = \dfrac{z^2 - z\cos\alpha}{z^2 - 2(\cos\alpha)z + 1}$. **8**

6. (a) Find $L\,f(t)$ where $f(t) = \cos t, \; 0 < t < \pi$ **6**

$\sin t \;\ge \pi$.

using the Heaviside unit step function.

(b) Find $z^{-1} \dfrac{1}{(z-2)(z-3)}$ for $2 < |z| < 3$ using residues. **6**

(c) Express the matrix $A = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ (\sin\alpha)(\sin\beta) & \cos\beta & -(\sin\beta)(\cos\alpha) \\ -(\cos\beta)(\sin\alpha) & \sin\beta & (\cos\alpha)(\cos\beta) \end{bmatrix}$ as a product of two orthogonal matrices. **8**

7. (a) Solve the DE $y + \int_0^t y\,dt = 1 - e^{-t}$ using laplace transforms. **6**

(b) If $0 \le x \le \pi$ show that– **6**

$$x^2 = \frac{2}{\pi}\left[\left(\frac{\pi^2}{1} - \frac{4}{1^3}\right)\sin x - \left(\frac{\pi^2}{2}\right)\sin 2x + \left(\frac{\pi^2}{3} - \frac{4}{3^3}\right)\sin 3x + \dots\right].$$

(c) Show that $\{\cos nx\}_{n \ge 1}$ is an orthogonal family of functions in $[-\pi, \pi]$. Also find the corresponding orthonormal set. **8**